

РАЗДЕЛ 1.

$$1) x^3 - 2x^2 - 9 = 0$$

Делители 9: - ±1: ±3: ±9

$$x = 3 \quad 27 - 18 - 9 = 0 \quad 0 = 0$$

$x = 3$ – корень уравнения

$$\begin{array}{r}
 \underline{x^3 - 2x^2 - 9} \quad \cdot \quad \left| \begin{array}{l} \underline{x - 3} \\ x^2 + x + 3 \end{array} \right. \\
 \underline{x^3 - 3x^2} \\
 x^2 - 9 \\
 \underline{-x^2 + 3x} \\
 3x - 9 \\
 \underline{-3x + 9} \\
 0
 \end{array}$$

$$(x^2 + x + 3)(x - 3) = 0$$

$$x^2 + x + 3 = 0$$

или

$$x - 3 = 0$$

$$D = 1 - 12 = -11 \text{ (корней нет)}$$

$$x = 3$$

Ответ: $x = 3$.

$$2) 6x^3 - x^2 - 20x + 12 = 0$$

$x_0 = \frac{p}{q}$ – несократимая дробь,

p – делитель 12 ±1: ±2: ±3: ±4: ±6: ±12

q – делитель 6 1; 2; 3; 6.

$$x = -2 \text{ – корень уравнения, т.к. } -48 - 4 + 40 + 12 = 0 \quad 0 = 0$$

$$\begin{array}{r}
 \underline{6x^3 - x^2 - 20x + 12} \quad \cdot \quad \left| \begin{array}{l} \underline{x + 2} \\ 6x^2 - 13x + 6 \end{array} \right. \\
 \underline{6x^3 + 12x^2} \\
 -13x^2 - 20x \\
 \underline{-13x^2 - 26x} \\
 6x + 12
 \end{array}$$

$$\begin{array}{r} 6x + 12 \\ - \underline{6x + 12} \\ 0 \end{array}$$

$$(6x^2 - 13x + 6)(x + 2) = 0$$

$$6x^2 - 13x + 6 = 0$$

или

$$x + 2 = 0$$

$$D = 169 - 144 = 25$$

$$x = -2$$

$$x_1 = \frac{13+5}{12} = 1,5$$

$$x_2 = \frac{13-5}{12} = \frac{2}{3}$$

Ответ: $x_1 = 1,5$; $x_2 = \frac{2}{3}$; $x_3 = -2$.

$$3) x^3 - 6x^2 + 5x + 12 = 0$$

Делители 12:

± 1 ; ± 2 ; ± 3 ; ± 4 ; ± 6 ; ± 12

$x = 1$ – корень уравнения т.к. $-1 - 6 - 5 + 12 = 0$

$$\begin{array}{r} x^3 - 6x^2 + 5x + 12 \\ - \underline{x^3 + x^2} \\ -7x^2 + 5x \\ - \underline{-7x^2 - 7x} \\ 12x + 12 \\ - \underline{12x + 12} \\ 0 \end{array} \quad \begin{array}{l} | x+1 \\ | x^2 - 7x + 12 \end{array}$$

$$(x^2 - 7x + 12)(x + 1) = 0$$

$$x^2 - 7x + 12 = 0$$

или

$$x + 1 = 0$$

$$x_1 + x_2 = 7$$

$$x_1 = 3$$

$$x = -1$$

$$x_1 \bullet x_2 = 12$$

$$x_2 = 4$$

Ответ: $x_1 = 3$; $x_2 = 4$; $x_3 = -1$.

$$4) (x - 1)^3 + (2x + 3)^3 = 27x^3 + 8$$

$$(x - 1 + 2x + 3)((x - 1)^2 - (x - 1)(2x + 3) + (2x + 3)^2) = (3x + 2)(9x^2 - 6x + 4)$$

$$(3x + 2)(x^2 - 2x + 1 - 2x^2 - 3x + 2x + 3 + 4x^2 + 12x + 9) = (3x + 2)(9x^2 - 6x + 4)(3x + 2)(3x^2 + 9x + 13) - (3x + 2)(9x^2 - 6x + 4) = 0$$

$$(3x^2 + 2)(3x^2 + 9x + 13 - 9x^2 + 6x - 4) = 0$$

$$3x + 2 = 0$$

или

$$-6x^2 + 15x + 9 = 0$$

$$x = -\frac{2}{3}$$

$$2x^2 - 5x - 3 = 0$$

$$D = 25 + 24 = 49$$

$$x_1 = \frac{5+7}{4} = 3$$

$$x_2 = \frac{5-7}{4} = -0,5$$

Ответ: $x_1 = \frac{2}{3}$; $x_2 = 3$; $x_3 = -0,5$

$$5) x^3 - (\sqrt{2} + 1)x^2 + 2 = 0$$

$$\sqrt{2} = a \quad 2 = a^2$$

$$x^3 - ax^2 - x^2 + a^2 = 0$$

$$a^2 - ax^2 + x^3 - x^2 = 0$$

$$D = x^2(x - 2)^2$$

$$a_{1,2} = \frac{x^2 \pm |x(x - 2)|}{2}$$

$$a_{1,2} = \frac{x^2 \pm x(x - 2)}{2}$$

$$a_1 = \frac{x^2 + x^2 - 2x}{2} = x^2 - x$$

$$a_2 = \frac{x^2 - x^2 + 2x}{2} = x$$

$$a_1 = x^2 - x$$

$$a_2 = x$$

$$x^2 - x = \sqrt{2}$$

$$x = \sqrt{2}$$

$$x^2 - x - \sqrt{2} = 0$$

$$D = 1 + 4\sqrt{2}$$

$$x_{1,2} = \frac{1 \pm \sqrt{1+4\sqrt{2}}}{2} -$$

$$\text{Ответ: } x = \frac{1 \pm \sqrt{1+4\sqrt{2}}}{2}; x = \sqrt{2}.$$

$$6) 9x^3 - 13x - 6 = 0$$

Делители - 6: $\pm 1 \pm 2 \pm 3 \pm 6$ (p)

Делители 9: 1; 3; 9 (q)

Возможные корни $\pm \frac{1}{3} \pm \frac{2}{3} \pm \frac{1}{9} \pm \frac{2}{9}$

$x = -\frac{2}{3}$ корень

$$\begin{array}{r} 9x^3 - 13x - 6 \\ \underline{9x^3 + 6x^2} \\ -6x^2 - 13x \\ \underline{-6x^2 - 4x} \\ -9x - 6 \\ \underline{-9x - 6} \\ 0 \end{array}$$

$$9x^2 - 6x - 9 = 0 \quad | :3$$

$$3x^2 - 2x - 3 = 0$$

$$D_1 = 1 + 9 = 10$$

$$x_{1,2} = \frac{1 \pm \sqrt{10}}{3}$$

$$\text{Ответ: } x = \frac{1 \pm \sqrt{10}}{3}; x = -\frac{2}{3}.$$

$$7) x^3 - 3x = 64 + \frac{1}{64}$$

$$64 + \frac{1}{64} = 4^3 + \left(\frac{1}{4}\right)^3 = 4^3 + 3 \cdot 4^2 \cdot \frac{1}{4} + 3 \cdot 4 \cdot \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 - 3 \cdot 4^2 \cdot \frac{1}{4} - 3 \cdot 4$$

$$\bullet \left(\frac{1}{4}\right)^2 = \left(4 + \frac{1}{4}\right)^3 - 3 \bullet 4 - 3 \bullet \frac{1}{4} = \left(4 + \frac{1}{4}\right)^3 - 3\left(4 + \frac{1}{4}\right)$$

$$x^3 - 3x = \left(4 + \frac{1}{4}\right)^3 - 3\left(4 + \frac{1}{4}\right)$$

$$x = 4 + \frac{1}{4} \text{ - корень}$$

$$\begin{array}{r|l} x^3 - 3x - 64\frac{1}{64} & x - 4\frac{1}{4} \\ x^3 - 4\frac{1}{4} & \hline & x^2 + 4\frac{1}{4}x + 15\frac{1}{16}x \end{array}$$

$$4\frac{1}{4}x^2 - 3x$$

$$4\frac{1}{4}x^2 - 18\frac{1}{16}x$$

$$15\frac{1}{16}x - 64\frac{1}{64}$$

$$\begin{array}{r} 15\frac{1}{16}x - 64\frac{1}{64} \\ \hline 0 \end{array}$$

$$x^2 + 4\frac{1}{4}x + 15\frac{1}{16}x = 0$$

$$x^2 + \frac{17}{4}x + \frac{241}{16} = 0 \quad | \bullet 16$$

$$16x^2 + 68x + 241 = 0$$

$$D_1 = (34)^2 - 16 \bullet 241 = 1156 - 3865 < 0 \quad (\text{корней нет})$$

$$\text{Ответ: } x = 4 + \frac{1}{4}.$$

$$\mathbf{8) \quad x(x+1) + (x+1)(x+2) + (x+2)(x+3) + (x+3)(x+4) + (x+4)(x+5) = 1 \bullet 2 + 2 \bullet 3 + 3 \bullet 4 + 4 \bullet 5}$$

Проверка показывает, что $x = 0$ и $x = -5$ являются корнями уравнения.

Так как после раскрытия скобок будет квадратное уравнение, то оно больше двух корней не может иметь, значит корни 0 и -5.

Ответ: 0; -5.

$$9) x^4 - (25 + \frac{1}{25})x^2 + 1 = 0$$

$$x^4 - 25x^2 - \frac{1}{25}x^2 + 1 = 0$$

$$x^2(x^2 - \frac{1}{25}) - 25(x^2 - \frac{1}{25}) = 0$$

$$(x^2 - \frac{1}{25})(x^2 - 25) = 0$$

$$x = \pm \frac{1}{5} \quad x = \pm 5$$

Ответ: $x = \pm \frac{1}{5}$; $x = \pm 5$.

$$10) x^4 + 2x^3 - x = 2$$

$$x^3(x+2) - (x+2) = 0$$

$$(x+2)(x^3-1) = 0$$

$$x = -2 \quad x = 1$$

Ответ: $x = -2$; $x = 1$.

$$11) x^4 + 3x^3 + 4x^2 + 6x + 4 = 0$$

$$(x^2 + 2)^2 + 3x^3(x^2 + 2) = 0$$

$$(x^2 + 2)(x^2 + 2 + 3x) = 0$$

$$x^2 + 3x + 2 = 0$$

$$x_1 + x_2 = -3 \quad x_1 = -1$$

$$x_1 \cdot x_2 = 2 \quad x_2 = -2$$

Ответ: $x = \pm 1$; $x = \pm 2$.

$$12) x^4 - 2x^3 - x^2 - 2x + 1 = 0 \quad | : x^2$$

$$x^2 - 2x - 1 - 2\frac{1}{x} + \frac{1}{x^2} = 0$$

$$x^2 + \frac{1}{x^2} - 2\left(x + \frac{1}{x}\right) - 1 = 0$$

$$x^2 + 2x \cdot \frac{1}{x} + \frac{1}{x^2} - 2 - 2\left(x + \frac{1}{x}\right) - 1 = 0$$

$$\left(x + \frac{1}{x}\right)^2 - 2\left(x + \frac{1}{x}\right) - 3 = 0$$

$$x + \frac{1}{x} = y$$

$$y^2 - 2y - 3 = 0$$

$$y_1 + y_2 = 2 \quad y_1 = 3$$

$$y_1 \cdot y_2 = -3 \quad y_2 = -1$$

$$x + \frac{1}{x} = 3 \quad | \cdot x \quad x + \frac{1}{x} = -1 \quad | \cdot x$$

$$x^2 - 3x + 1 = 0 \quad x^2 + x + 1 = 0$$

$$D = 9 - 4 = 5 \quad D = 1 - 4 < 0 \quad (\text{корней нет})$$

$$x_{1,2} = \frac{3 \pm \sqrt{5}}{2}$$

$$\text{Ответ: } x = \frac{3 \pm \sqrt{5}}{2}.$$

$$\mathbf{13) \quad x^5 - 2x^4 - 6x^3 + 12x^2 + x - 2 = 0}$$

$$x^4(x - 2) - 6x^2(x - 2) + (x - 2) = 0$$

$$(x - 2)(x^4 - 6x^2 + 1) = 0$$

$$x = 2 \quad \text{или} \quad x^4 - 6x^2 + 1 = 0$$

$$x^2 = y > 0$$

$$y^2 - 6y + 1 = 0$$

$$D_1 = 9 - 1 = 8$$

$$y_1 = 3 \pm \sqrt{8}$$

$$x^2 = 3 + \sqrt{8}$$

$$x^2 = 3 - \sqrt{8}$$

$$x_{1,2} = \pm \sqrt{3 + \sqrt{8}}$$

$$x_{3,4} = \pm \sqrt{3 - \sqrt{8}}$$

$$\text{Ответ: } x = \pm \sqrt{3 + \sqrt{8}}; \quad x = \pm \sqrt{3 - \sqrt{8}}.$$

$$14) x^6(1-x) - x^3(1-x^2) + x - x^2 = 0$$

$$x^6(1-x) - x^3(1+x) + x(1-x) = 0$$

$$(1-x)(x^6 - x^3(1+x) + x) = 0$$

$$1-x=0 \quad x^6 - x^3(1+x) + x = 0$$

$$x=1 \quad x(x^5 - x^2(1+x) + 1) = 0$$

$$x^3(x^2 - 1) - (x^2 - 1) = 0$$

$$(x^2 - 1)(x^3 - 1) = 0$$

$$(x-1)(x+1)(x-1)(x^2+x+1) = 0$$

$$x = \pm 1$$

Ответ: $x = 0$; $x = \pm 1$.

РАЗДЕЛ 2

№1

$$\frac{4x}{4x^2 - 8x + 7} + \frac{3x}{4x^2 - 10x + 7} = 1$$

Так как $x=0$ не является корнем уравнения, то разделим и числитель и знаменатель на x .

$$\frac{4}{4x - 8 + \frac{7}{x}} + \frac{3}{4x - 10 + \frac{7}{x}} = 1$$

$$4x + \frac{7}{x} = a$$

$$\frac{4}{a-8} + \frac{3}{a-10} = 1 \quad |(a-8)(a-10) \neq 0$$

$$4(a-10) + 3(a-8) = a^2 - 18a + 80$$

$$4a - 40 + 3a - 24 = a^2 - 18a + 80$$

$$a^2 - 25a + 144 = 0$$

$$D = 625 - 576 = 49$$

$$a_1 = \frac{25+7}{2} = 16$$

$$a_2 = \frac{25-7}{2} = 9$$

$$1) 4x + \frac{7}{x} = 16 \quad | \cdot x$$

$$4x^2 - 16x + 7 = 0$$

$$D_1 = 64 - 28 = 36$$

$$x_1 = \frac{8+6}{4} = 3,5$$

$$x_2 = \frac{8-6}{4} = \frac{1}{2}$$

$$2) 4x + \frac{7}{x} = 9 \quad | \cdot x$$

$$4x^2 - 9x + 7 = 0$$

$$D = 81 - 4 \cdot 4 \cdot 7 = -31$$

$$D < 0$$

Корней нет.

Ответ: $x_1 = 3,5; x_2 = \frac{1}{2}$

№2

$$x^4 - 2x^3 - x^2 - 2x + 1 = 0 \mid \div x^2$$

$$x^2 - 2x - 1 - \frac{2}{x} + \frac{1}{x^2} = 0$$

$$\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) - 1 = 0$$

$$\left(x + \frac{1}{x}\right)^2 = x^2 + 2 + \frac{1}{x^2} = x^2 + \frac{1}{x^2} + 2$$

$$x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$$

$$\left(x + \frac{1}{x}\right)^2 - 2 - 2\left(x + \frac{1}{x}\right) - 1 = 0$$

$$x + \frac{1}{x} = y$$

$$y^2 - 2y - 3 = 0$$

$$y_1 + y_2 = 2$$

$$y_1 y_2 = -3$$

$$y_1 = 3$$

$$y_2 = -1$$

$$1) x + \frac{1}{x} = 3 \mid \cdot x$$

$$x^2 - 3x + 1 = 0$$

$$D = 9 - 4 = 5$$

$$x = \frac{3 \pm \sqrt{5}}{2}$$

$$2) x + \frac{1}{x} = -1 \mid \cdot x$$

$$x^2 + x + 1 = 0$$

$$D = 1 - 4 = -3$$

$$D < 0$$

Корней нет.

$$\text{Ответ: } x = \frac{3 \pm \sqrt{5}}{2}$$

№3

$$(x^2 - x + 1)^4 - 8x^2(x^2 - x + 1)^2 + 16x^4 = 0$$

$$(x^2 - x + 1)^4 - 8x^2(x^2 - x + 1)^2 + 16x^4 = 0 \mid \div x^4$$

$$\left(x - 1 + \frac{1}{x}\right)^4 - 8\left(x - 1 + \frac{1}{x}\right)^2 + 16 = 0$$

$$\left(x - 1 + \frac{1}{x}\right)^2 = t$$

$$t^2 - 8t + 16 = 0$$

$$(t - 4)^2 = 0$$

$$t = 4$$

$$x - 1 + \frac{1}{x} = \pm 2$$

$$1) x + \frac{1}{x} - 1 = 2$$

$$x + \frac{1}{x} - 3 = 0 \mid \cdot x$$

$$x^2 - 3x + 1 = 0$$

$$D = 9 - 4 = 5$$

$$x_1 = \frac{3 \pm \sqrt{5}}{2}$$

$$2) x + \frac{1}{x} - 1 = -2$$

$$x + \frac{1}{x} + 1 = 0 \mid \cdot x$$

$$x^2 + x + 1 = 0$$

$$D < 0$$

решений нет.

$$\text{Ответ: } x_1 = \frac{3 \pm \sqrt{5}}{2}.$$

№4

$$\frac{1}{2x^2 - x + 1} + \frac{3}{2x^2 - x + 3} = \frac{10}{2x^2 - x + 7}$$

$$y = 2x^2 - x + 4$$

$$2x^2 - x + 1 = y - 3$$

$$2x^2 - x + 7 = y + 3$$

$$2x^2 - x + 3 = y - 1$$

$$\frac{1}{y-3} + \frac{3}{y-1} - \frac{10}{y+3} = 0 \cdot (y-3)(y-1)(y+3) \neq 0$$

$$(y-1)(y+3) + 3(y-3)(y+3) - 10(y-3)(y-1) = 0$$

$$y^2 + 2y - 3 + 3y^2 - 27 - 10y^2 + 40y - 30 = 0$$

$$-6y^2 + 42y - 60 = 0 \quad | : -6$$

$$y^2 - 7y + 10 = 0$$

$$y_1 = 2$$

$$y_2 = 5$$

$$1) 2x^2 - x + 4 = 2$$

$$2x^2 - x + 2 = 0$$

$$D = 1 - 16 = -15$$

$$D < 0$$

Решений нет.

$$2) 2x^2 - x + 4 = 5$$

$$2x^2 - x - 1 = 0$$

$$D = 1 + 8 = 9$$

$$x_1 = \frac{1+3}{4} = 1$$

$$x_2 = \frac{1-3}{4} = -0,5$$

Ответ: $x_1 = 1; x_2 = -0,5$

№5

$$\frac{x^2 + x + 1}{x^2 - x + 1} = \frac{7(x+1)}{9(x-1)} \cdot \frac{x-1}{x+1} \neq 0$$

$$\frac{x^3 - 1}{x^3 + 1} = \frac{7}{9}$$

$$9x^3 - 9 = 7x^3 + 7$$

$$2x^3 = 16$$

$$x^3 = 8$$

$$x = 2$$

Ответ: $x=2$.

№6

$$\frac{x-1}{x+2} + \frac{x+2}{x-1} = \frac{x+5}{x+3} + \frac{x+3}{x+5}$$

$$\frac{x+2-3}{x+2} + \frac{x-1+3}{x-1} = \frac{x+3+2}{x+3} + \frac{x+5-2}{x+5}$$

$$1 - \frac{3}{x+2} + 1 + \frac{3}{x-1} = 1 + \frac{2}{x+3} + 1 - \frac{2}{x+5}$$

$$\frac{3}{x-1} - \frac{3}{x+2} = \frac{2}{x+3} - \frac{2}{x+5}$$

$$3 \left(\frac{x+2-x+1}{(x-1)(x+2)} \right) = 2 \left(\frac{x+5-x-3}{(x+3)(x+5)} \right)$$

$$\frac{9}{(x-1)(x+2)} = \frac{4}{(x+3)(x+5)}$$

$$9(x^2 + 8x + 15) = 4(x^2 + x - 2)$$

$$9x^2 + 72x + 135 = 4x^2 + 4x - 8$$

$$5x^2 + 68x + 143 = 0$$

$$D_1 = 1156 - 715 = 441$$

$$x_1 = \frac{-34 + \sqrt{441}}{5} = \frac{-34 + 21}{5} = -\frac{13}{5}$$

$$x_2 = \frac{-34 - 21}{5} = \frac{-55}{5} = -11$$

Ответ: $x_1 = -\frac{13}{5}; x_2 = -11$.

№7

$$x^4 - 8x^3 - 4x^2 + 16x + 4 = 0$$

$$(x^2 - 2)^2 - 8x(x^2 - 2) = 0$$

$$(x^2 - 2)(x^2 - 2 - 8x) = 0$$

$$x^2 - 8x - 2 = 0$$

$$x = \pm\sqrt{2}$$

$$D_1 = 16 + 2 = 18$$

$$x = 4 \pm \sqrt{18} = 4 \pm 3\sqrt{2}$$

ОТВЕТ: $x = \pm\sqrt{2}; 4 \pm 3\sqrt{2}$

№8

$$(x^2 - 6x - 9)^2 = x^3 - 4x^2 - 9x$$

$$(x^2 - 6x - 9)^2 = x(x^2 - 4x - 9) \div x^2$$

$$\left(x - 6 - \frac{9}{x}\right)^2 = \left(x - 4 - \frac{9}{x}\right)$$

$$x - \frac{9}{x} - 4 = a$$

$$(a - 2)^2 = a$$

$$a^2 - 4a + 4 - a = 0$$

$$a^2 - 5a + 4 = 0$$

$$a_1 + a_2 = 5$$

$$a_1 a_2 = 4$$

$$a_1 = 1 \quad a_2 = 4$$

$$1) x - \frac{9}{x} - 4 = 1 \cdot x$$

$$x^2 - 5x - 9 = 0$$

$$D = 25 - 4(-9) = 25 + 36 = 61$$

$$x = \frac{5 \pm \sqrt{61}}{2}$$

$$2) x - \frac{9}{x} - 4 = 4$$

$$x - \frac{9}{x} - 8 = 0 \mid x$$

$$x^2 - 8x - 9 = 0$$

$$x_1 + x_2 = 8$$

$$x_1 x_2 = -9$$

$$x_1 = 9 \quad x_2 = -1$$

ОТВЕТ: $x = \frac{5 \pm \sqrt{61}}{2}; 9; -1.$

№

$$\left(3x + \frac{1}{x}\right)^2 - 6 - 2\left(3x + \frac{1}{x}\right) - 18 = 0$$

$$\left(3x + \frac{1}{x}\right) = a$$

$$a^2 - 2a - 24 = 0$$

$$a_1 = 6$$

$$a_2 = 4$$

$$1) 3x + \frac{1}{x} = 6 \mid \cdot x$$

$$3x^2 - 6x + 1 = 0$$

$$D_1 = 9 - 3 = 6$$

$$x_1 = \frac{3 \pm \sqrt{6}}{3}$$

$$2) 3x + \frac{1}{x} + 4 = 0 \mid \cdot x$$

$$3x^2 + 4x + 1 = 0$$

$$D_1 = 4 - 3 = 1$$

$$x_1 = -1$$

$$x_2 = -\frac{1}{3}$$

$$\text{Ответ: } x = \frac{3 \pm \sqrt{6}}{3}; -1; -\frac{1}{3}$$

№9

$$(x+2)(x+3)(x+8)(x+12) = 4x^2$$

$$(x^2 + 14x + 24)(x^2 + 11x + 24) = 4x^2 \mid \div x^2$$

$$\left(x + \frac{24}{x} + 14\right)\left(x + \frac{24}{x} + 11\right) = 4$$

$$x + \frac{24}{x} = y$$

$$(y+14)(y+11) = 4$$

$$y^2 + 25y + 154 - 4 = 0$$

$$y^2 + 25y + 150 = 0$$

$$D = 625 - 600 = 25$$

$$y_1 = \frac{-25 + 5}{2} = -10$$

$$y_2 = \frac{-25 - 5}{2} = -15$$

$$1) x + \frac{24}{x} = -10 \mid \cdot x$$

$$x^2 + 10x + 24 = 0$$

$$x_1 + x_2 = -10$$

$$x_1 x_2 = 24$$

$$x_1 = -6$$

$$x_2 = -4$$

$$2) x + \frac{24}{x} = -15 \mid \cdot x$$

$$x^2 + 15x + 24 = 0$$

$$x_1 + x_2 = -15$$

$$x_1 x_2 = 24$$

$$D = 225 - 96 = 129$$

$$x_1 = \frac{-15 \pm \sqrt{129}}{2}$$

$$\text{Ответ: } x = \frac{-15 \pm \sqrt{129}}{2}; -6; -4.$$

№10

$$(x-1)(x-2)(x-4)(x-5) = 7(x-3)^2$$

$$x-3 = a$$

$$x-1 = a+2; x-2 = a+1; x-4 = a-1; x-5 = a-2;$$

$$(a+2)(a+1)(a-1)(a-2) = 7a^2$$

$$(a^2 + 3a + 2)(a^2 - 3a + 2) = 7a^2 \quad | : a^2$$

$$\left(a + 3 + \frac{2}{a}\right)\left(a - 3 + \frac{2}{a}\right) = 7$$

$$a + \frac{2}{a} = t$$

$$(t+3)(t-3) = 7$$

$$t^2 - 9 = 7$$

$$t^2 = 16$$

$$t = \pm 4$$

$$1) a + \frac{2}{a} = 4 \quad | \times a$$

$$a^2 - 4a + 2 = 0$$

$$D_1 = 4 - 2 = 2$$

$$a_1 = 2 \pm \sqrt{2}$$

$$x = 5 \pm \sqrt{2}$$

$$2) a + \frac{2}{a} = -4 \quad | \cdot 9$$

$$a^2 + 4a + 2 = 0$$

$$D_1 = 4 - 2 = 2$$

$$a_1 = -2 \pm \sqrt{2}$$

$$x - 3 = -2 \pm \sqrt{2}$$

$$x = 1 \pm \sqrt{2}$$

$$\text{Ответ: } x_1 = 5 \pm \sqrt{2}; x_2 = 1 \pm \sqrt{2}$$

№11

$$\frac{x^2 + 2x + 2}{x+1} + \frac{x^2 + 8x + 20}{x+4} = \frac{x^2 + 4x + 6}{x+2} + \frac{x^2 + 6x + 12}{x+3}$$

$$\frac{x^2 + x + x + 2}{x+1} + \frac{x^2 + 4x + 4x + 20}{x+4} = \frac{x^2 + 2x + 2x + 6}{x+2} + \frac{x^2 + 3x + 3x + 12}{x+3}$$

$$x + \frac{x+2}{x+1} + x + \frac{4x+20}{x+4} = x + \frac{2x+6}{x+2} + x + \frac{3x+12}{x+3}$$

$$\frac{x+1+1}{x+1} + \frac{4x+16+4}{x+4} = \frac{2x+4+2}{x+2} + \frac{3x+9+3}{x+3}$$

$$1 + \frac{1}{x+1} + 4 + \frac{4}{x+4} = 2 + \frac{2}{x+2} + 3 + \frac{3}{x+3}$$

$$\frac{1}{x+1} + \frac{4}{x+4} = \frac{2}{x+2} + \frac{3}{x+3}$$

$$\frac{x+4+4x+4}{x^2+5x+4} = \frac{2x+6+3x+6}{x^2+5x+6}$$

$$\frac{5x+8}{x^2+5x+4} = \frac{5x+12}{x^2+5x+6}$$

$$5x^3 + 25x^2 + 30x + 8x^2 + 40x + 48 - 5x^3 - 25x^2 - 20x - 12x^2$$

$$-4x^2 - 10x = 0$$

$$-x(4x+10) = 0$$

$$4x+10 = 0$$

$$x = 0 \quad \text{или} \quad x = \frac{5}{2}$$

$$\text{Ответ: } x = \frac{5}{2}; 0.$$

№12

$$(x^3 + x^2 + 1)^2 + (x^3 - x^2 + 1)^2 = 2x^4 \mid \div x^4$$

$$\left(x+1+\frac{1}{x^2}\right)^2 + \left(x-1+\frac{1}{x^2}\right)^2 = 2$$

$$x + \frac{1}{x^2} = a$$

$$(a+1)^2 + (a-1)^2 = 2$$

$$a^2 + 2a + 1 + a^2 - 2a + 1 = 2$$

$$a^2 = 0$$

$$x + \frac{1}{x^2} = 0 \mid \div x^2$$

$$x^3 + 1 = 0$$

$$x = -1$$

$$\text{Ответ: } x = -1$$

№13

$$x^3 + \frac{1}{x^3} = 6\left(x + \frac{1}{x}\right)$$

$$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$$

$$\left(x + \frac{1}{x}\right)^3 - 9\left(x + \frac{1}{x}\right) = 0$$

$$\left(x + \frac{1}{x}\right)\left(\left(x + \frac{1}{x}\right)^2 - 9\right) = 0$$

$$x + \frac{1}{x} = 0 \quad \text{или} \quad \left(x + \frac{1}{x}\right) = \pm 3$$

$$1) x + \frac{1}{x} = 0 \mid \cdot x$$

$$x^2 + 1 = 0$$

Решений нет.

$$2) x + \frac{1}{x} = \pm 3$$

$$x + \frac{1}{x} = 3 \mid \cdot x$$

$$x^2 - 3x + 1 = 0$$

$$D = 9 - 4 = 5$$

$$x_1 = \frac{3 \pm \sqrt{5}}{2}$$

$$x + \frac{1}{x} = -3 \mid \cdot x$$

$$x^2 + 3x + 1 = 0$$

$$D = 9 - 4 = 5$$

$$x_2 = \frac{-3 \pm \sqrt{5}}{2}$$

$$\text{Ответ: } x_1 = \frac{3 \pm \sqrt{5}}{2}; x_2 = \frac{-3 \pm \sqrt{5}}{2}$$

№14

$$(x^2 + 3x + 4)(x^2 + 4x + 5) = 2(x+1)^2$$

$$(x^2 + 2x + 1 + x + 3)(x^2 + 2x + 1 + 2x + 4) = 2(x+1)^2$$

$$((x+1)^2 + (x+1) + 2)((x+1)^2 + 2(x+1+1)) = 2(x+1)^2$$

$$x+1 = a$$

$$(a^2 + a + 2)(a^2 + 2a + 2) = 2a^2 \mid \div a^2$$

$$\left(a + 1 + \frac{2}{a}\right)\left(a + 2 + \frac{2}{a}\right) = 2$$

$$\left(a + \frac{2}{a}\right) = t$$

$$(t+1)(t+2) = 2$$

$$t^2 + 3t + 2 = 2$$

$$t_1 = 0$$

$$t_2 = -3$$

$$1) a + \frac{2}{a} = 0 \mid \cdot a$$

$$a^2 + 2 = 0$$

Решений нет.

$$2) a + \frac{2}{a} + 3 = 0 \mid \cdot a$$

$$a^2 + 3a + 2 = 0$$

$$a_1 + a_2 = -3$$

$$a_1 a_2 = 2$$

$$a_1 = -2$$

$$a_2 = -1$$

$$1) x+1 = -2$$

$$x = -3$$

$$2) x+1 = -1$$

$$x = -2$$

Ответ: $x = -3; -2$

№15

$$x^2 + \left(\frac{2x}{x-2}\right)^2 = 5$$

Пусть x_0 - решение уравнения, тогда пусть

$$y_0 = \frac{2x_0}{x_0 - 2}, \text{ тогда}$$

$$y_0 x_0 - 2y_0 - 2x_0 = 0$$

$$2(x_0 + y_0) - x_0 y_0 = 0$$

$$\begin{cases} x_0^2 + y_0^2 = 5 \\ 2(x_0 + y_0) - x_0 y_0 = 0 \end{cases}$$

$$\begin{cases} (x_0 + y_0)^2 - 2x_0 y_0 = 5 \\ 2(x_0 + y_0) - x_0 y_0 = 0 \end{cases}$$

$$x_0 + y_0 = u$$

$$x_0 y_0 = v$$

$$\begin{cases} u^2 - 2v = 5 \\ 2u - v = 0 \end{cases}$$

$$\begin{cases} v = 2u \\ u^2 - 4u - 5 = 0 \end{cases}$$

$$\begin{cases} u_1 + u_2 = 4 \\ u_1 u_2 = -5 \end{cases}$$

$$u_1 = 5; u_2 = -1$$

$$v_1 = 10; v_2 = -2$$

$$1) \begin{cases} x_0 + y_0 = 5 \\ x_0 y_0 = 10 \end{cases}$$

$$y_0 = 5 - x_0$$

$$x_0(5 - x_0) = 10$$

$$5x_0 - x_0^2 - 10 = 0$$

$$x_0^2 - 5x_0 + 10 = 0$$

$$D = 25 - 40 = -15$$

$$D < 0$$

Решений нет.

$$2) \begin{cases} x_0 + y_0 = -1 \\ x_0 y_0 = -2 \end{cases}$$

$$x_0 = -2; y_0 = 1$$

$$y_0 = -2; x_0 = 1$$

Ответ: $x = 1; -2$